

WHITNEY WORKSHOP SCHEDULE
College of William and Mary, August 11-16, 2008
All meetings are in Jones 131

Monday, August 11

8:30 – 9:45 Setting (computer) accounts, etc.

9:45 – 10:00 Opening

10:00 – 11:00 **Fefferman** Lecture 1: Survey

Abstract: This series of talks gives an overview of (what I know about) extension and interpolation, and discusses some open problems.

11:15 – 12:15 **Fefferman** Lecture 2: Survey

12:15 – 2:45 Lunch break

2:45 – 3:45 **Fefferman** Lecture 3: Survey

4:00 – 5:00 **Fefferman** Lecture 4: Survey

Tuesday, August 12

9:00 – 10:00 **Shvartsman** Lecture 1: "Sobolev spaces on closed subsets and domains in \mathbb{R}^n "

Abstract: We give an intrinsic characterization of the restrictions of Sobolev spaces $W_p^1(\mathbb{R}^n)$, $p > n$, to an arbitrary closed subset S in \mathbb{R}^n via local oscillations and certain doubling measures supported on S . We also prove an extension theorem for Sobolev functions defined on a class of subhyperbolic domains in \mathbb{R}^n . For $p > 2$ this enables us to characterize simply connected Sobolev W_p^1 -extension domains in \mathbb{R}^2 .

10:15 – 11:15 **Shvartsman** Lecture 2: "Sobolev spaces on closed subsets and domains in \mathbb{R}^n "

11:30 – 12:30 **Shvartsman** Lecture 3: "Sobolev spaces on closed subsets and domains in \mathbb{R}^n "

12:30 – 2:45 Lunch break

2:45 – 3:45 **Fefferman** Lecture 1: " C^m Extensions with Norm Arbitrarily Close to Least Possible"

Abstract: Fix m, n . Let f be a real-valued function on an N -point set E in \mathbb{R}^n . Given ϵ positive, one wants to compute an extension F of f , whose C^m -norm is within ϵ percent of least possible. A counterexample (joint work with Bo'az) shows that the usual finiteness principle fails in this setting. The talk gives some conjectures and partial results. In particular, as a substitute for the finiteness principle, we discuss "moves".

4:00 – 5:00 **Fefferman** Lecture 2: " C^m Extensions with Norm Arbitrarily Close to Least Possible"

Wednesday, August 13

9:00 – 10:00 **Yomdin** Lecture 1 "Discrete Remez Inequality"

Abstract: The classical Remez inequality bounds the maximum of the absolute value of a polynomial $P(x)$ of degree d on $[-1, 1]$ through the maximum of its absolute value on any subset Z of positive measure in $[-1, 1]$. Similarly, in several variables the maximum of the absolute value of a polynomial $P(x)$ of degree d on the unit ball $B_1^n \subset \mathbb{R}^n$ can be bounded through the maximum of its absolute value on any subset $Z \subset B_1^n$ of positive n -measure. The main result we present is that the n -measure in the Remez inequality can be replaced by a certain geometric invariant $\omega_d(Z)$ which can be effectively estimated in terms of the metric entropy of Z and which may be nonzero for discrete and even finite sets Z .

10:15 – 11:15 **Yomdin** Lecture 2: "Discrete Remez Inequality"

11:30 – 12:30 **Yomdin** Lecture 3: "Discrete Remez Inequality"

12:30 – 2:45 Lunch break

2:45 – 3:45 **Brudnyi** Lecture 1: "Characterization of Traces of Certain Morrey-Campanato Spaces to Regular Subsets of \mathbb{R}^n ".

Abstract: We present a characterization of traces of Morrey-Campanato spaces defined by quasipower majorants to Ahlfors regular subsets of \mathbb{R}^n . For particular choices of the majorant these spaces coincide with some Besov, Sobolev, Holder and Marchaud-Zygmund spaces (joint work with Yu. Brudnyi).

4:00 – 5:00 **Brudnyi** Lecture 2: "Characterization of Traces of Certain Morrey-Campanato Spaces to Regular Subsets of \mathbb{R}^n ".

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Thursday, August 14

9:00 – 10:00 **Zobin** Lecture 1: "Dualities in extension problems"

10:15 – 11:15 **Zobin** Lecture 2: "Dualities in extension problems"

11:30 – 12:30 **Zobin** Lecture 3: "Dualities in extension problems"

12:30 – 2:45 Lunch break

2:45 – 3:45 Problem/discussion session

4:00 – 5:00 Problem/discussion session

Friday, August 15

9:00 – 10:00 **Shvartsman** Lecture 1: "Extensions of smooth functions and Lipschitz selections of set-valued mappings"

Abstract: We will discuss interesting connections of the Whitney extension problem and combinatorial geometry of convex bodies in \mathbb{R}^n . In particular, we will see that the Whitney problem for the space $C^{1,1}(\mathbb{R})$ is equivalent to a problem of the existence of Lipschitz selections of certain set-valued mappings in \mathbb{R} . We will present a Helly-type criterion for the existence of such selections and will discuss the connections of this criterion with the phenomenon of finiteness in the Whitney problem. We show that this criterion is also closely connected with the problem of the existence of a Lipschitz (with respect to the Hausdorff distance) selector defined on the family of all convex finite dimensional compacts in a Banach space X . If X is a Hilbert space, then the classical Steiner point of a convex body provides such a selector, but in the non-Hilbert case there is no known way of constructing such a point. We prove the existence of the required Lipschitz continuous selector for an arbitrary Banach space X . The proof is based on a new result about Lipschitz properties of the center of mass of a convex set. We will also show how the Whitney extension problem for the space $C^{k,\omega}(\mathbb{R})$ and the Zygmund space $Z_m(\mathbb{R})$ can be reformulated as Lipschitz selection problems for certain classes of set-valued mappings defined on \mathbb{R}^n and finite-dimensional hyperbolic space.

10:15 – 11:15 **Shvartsman** Lecture 2 "Extensions of smooth functions and Lipschitz selections of set-valued mappings"

11:30 – 12:30 **Shvartsman** Lecture 3 "Extensions of smooth functions and Lipschitz selections of set-valued mappings"

12:30 – 2:45 Lunch break

2:45 – 3:45 Problem/discussion session

4:00 – 5:00 Problem/discussion session

Saturday, August 16

9:00 – 10:00 **Fefferman** Concluding remarks

10:15 – 11:15 Problem/discussion session

11:30 – 12:30 Problem/discussion session

12:30 – 2:45 Lunch break

2:45 – 3:45 Problem/discussion session

4:00 – 5:00 Problem/discussion session